Journal of Statistical Physics, Vol. 71, Nos. 5/6, 1993

Book Review: *Fractals and Disordered Systems*

Fractals and Disordered Systems. Armin Bunde and Shlomo Havlin, eds., Springer-Verlag, Berlin, 1991.

This book might be considered a delayed response to Leo Kadanoff's essay of 6 years ago entitled,[1] "Fractals: Where's the physics?" *Fractals and Disordered Systems* is a useful and well-integrated selection of ten chapters describing theoretical and experimental progress over the past 5–10 years in fractals and the physics of disordered systems. As the editors (who are also the authors of Chapters 2 and 3 on percolation) point out in the Preface, the fractals field is developing at a very rapid pace, and this book is intended to fill the gap between advanced textbooks and the current journal literature. This pace of progress is possible because the theoretical needs cited by Kadanoff are being addressed.

The chapters are written by 12 well-recognized experts who provide relevant background information for the reader who choses to peruse them out of order, but the authors have also taken care to standardize notations and cite cross references to sections of other chapters which complement their own. (Aharony mentions in Chapter 4 that in all other of his writings the fractal dimension d_f is designated D.) The chapters and authors are:

1. Fractals and Multifractals: The Interplay of Physics and Geometry, H. Eugene Stanley
2. Percolation I, A. Bunde and S. Havlin
3. Percolation II, S. Havlin and A. Bunde
4. Fractal Growth, A. Aharony
5. Fractures, H. J. Hermann
6. Fractals Electrodes, Fractal Membranes, and Fractal Catalysts, B. Sapoval

[1] L. P. Kadanoff, *Physics Today* **1986** (February):6–7.

0022-4715/93/0600-1233$07.00/0

7. Fractal Surfaces and Interfaces, J. F. Gouyet, M. Rosso, and B. Sapoval

8. Fractals and Experiments, J. K. Kjems

9. Cellular Automata, D. Stauffer

10. Exactly Self-Similar Left-Sided Multifractals, B. B. Mandelbrot and C. J. G. Evertsz

The first four chapters comprise a coherent overview of the theory of statistically self-similar (i.e., fractal) structures, emphasizing in particular how the fractal geometry of the structures is (or may be) related to the physics of disorder. The primary subjects of these chapters are three well-known models of disorder which have a clear relevance to physics: the random walk, percolation, and diffusion-limited aggregation (DLA).

In Chapter 1, Fractals and Multifractals, Stanley uses familiar examples to review the key structural and scaling features of nonrandom and random fractals [i.e., objects whose "mass" density $p(L)$ decreases as a power law with increasing size L]. This is followed by a discussion of DLA as a prototypical fractal whose surprisingly rich structure results from a very simple growth mechanism. The realization of DLA in nearly 50 physical systems is cited.

While the ubiquity of DLA structures in such widely disparate phenomena as coagulation, electrodeposition, electrical and mechanical breakdown, and dendritic growth is convincingly demonstrated to be linked to Laplace's equation, a simple interpretation of the broad distribution of growth probabilities is still not available. The void-channel model of DLA growth presented is a promising step toward this goal. Stanley's discussion of the way channel tips grow together to shield each other and thus limit further growth illustrates how physical intuition plays a major role in the development of theory. To compensate the reader (or reviewer) who has not yet mastered the ten or more distinct fractal dimensions necessary to describe DLA and percolation, this introductory chapter furnishes examples of unsolved research problems and conveys the sense of discovery and excitement that the field holds.

In the next two chapters Bunde and Havlin discuss static (Chapter 2) and dynamic (Chapter 3) features of the percolation model. First introduced by Flory and Stockmayer 50 years ago to discuss polymer gelation, the percolation model is the best-understood example of a geometrical phase transition. The strong connection between statistical self-similarity, fluctuations, and phase transitions is highlighted by the exact relation between the fractal dimension of the infinite percolation cluster d_f and the critical exponents of the percolation transition. The Cayley tree cluster size distribution is characterized by two scaling exponents. By analogy, a scaling theory is developed to extend these results to arbitrary percolation geometries. The treatment of dynamics in the percolation system (i.e., finite and/or infinite clusters) is particularly impressive as regards the wealth of electrical, mechanical, and transport applications and examples discussed in terms of the same ten or so scaling exponents (not all independent!). Topics such as the "mixed alkali effect" in vitreous oxide glasses and biased fields slowing down random walker motion illustrate the surprising richness and future potential of the percolation model.

Chapter 4 summarizes the less-well-understood subject of growth models and mechanisms which lead to DLA and percolation structures. Aharony presents the dielectric breakdown model (DBM) as a generalization of the DLA algorithm, with slightly modified boundary conditions. When the bond probabilities in DBM are independent of field strength, the model reduces to the Eden model and leads to compact (nonfractal) geometries. While a complete theory of the growth process is still a distant goal, it is clear that the main issues have been identified.

The next four chapters describe applications to fracture, fractal surfaces such as electrodes and membranes, as well as the experimental methods used to characterize such systems.

To summarize, this is an excellent collection of articles describing many recent developments in the field of fractal physics, and goes a long way toward answering Kadanoff's question.

John T. Bendler
General Electric Corporate
Research and Development
Schenectady, New York